S. Piplaya.

SHRI VAISHNAV INSTITUTE OF TECHNOLOGY & SCIENCE, BAROLI INDORE

Unit Test No.	Date	Code No. / Subject	Roll No. & Name		Marks Obtained
		Machine Design.		Spring. Notes:	Maximum Marks

Spring:

Elastic body, whose function is to distort when loaded & recover its original shape when the load is removed.

Applications: To Cushion, shock absorb, vibration dampers. watches, toys etc

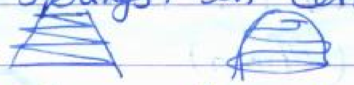
Types of Springs

i) Helical: a) Compression b) Tension

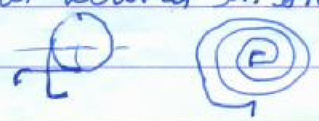
closely coiled - helix angle is very small, less than 10°.

Open Coiled - - " ~~-~~ Large, Greater than 10°

ii) Conical & Volute Springs: in cone shaped Comp. Spring



iii) ~~Spiral Spring~~ Torsion spring: Flat wound in the form of spring & loaded in torsion



iv) Leaf Spring

v) Disc Spring.

vi) Special Purpose Spring.

Materials:- High fatigue strength, high ductility, high resilience, & creep resist

i) Carbon from 0.5 to 1 % C

ii) Alloy steel - for high load, & repeated load

iii) Spring Brass, Phosphor Bronze, & Monel-Metal are used when corrosion resistant spring are needed.

iv) 55 Si 2 Mn 90 & 50 Cr 1 V23 are commonly used in making springs.

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Terminology used in Spring:

a. Spring Index $C = D/d$ mean dia. of coil / dia of wire

b. Solid Length $\underline{\underline{L_s}} = n'd$
n' - Total no. of coils
d - wire dia

c. Free Length $L_F = L_s$ + Max. Comp. + clearance b/w adjacent coil (Clash Allowance)
$$L_F = n'd + \delta_{max} + 0.15\,\delta_{max}$$
or
$$L_F \Rightarrow n'd + \delta_{max} + (n'-1) \times 1\text{ mm}.$$

d. Deflection: It is the distance moved by spring under the action of load (δ)
$$\delta = \frac{\text{Load}}{\text{Stiffness}} = \frac{W}{K} \text{ (mm)}$$

e. Stiffness: It is defined as the ratio of load to deflection
or spring Rate
$$K = \frac{W}{\delta} \text{ (N/mm)}$$

f. Shear stress factor: K_s
$$K_s = 1 + \frac{1}{2C}$$

g. Wahl's correction factor: K_w (takes care of curvature of springs)
$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

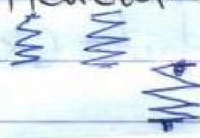
h. Pitch: The is axial distance b/w adjacent coils in uncompressed state: $p = \frac{L_F}{(n'-1)}$

$$p = \frac{L_F - L_s}{n'} + d$$

i. For Squared & Ground Ends:
Total no. of coils = no. of Active coils + 2

j. Active Coils: take part in deflection

- End Connections of Helical Comp. Springs
  - Plain Ends
  - Ground
  - Squared
  - Squared & Ground

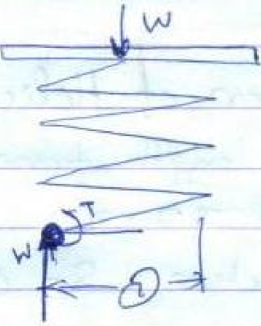


- End ~~Condi~~ Connections for Tension Helical spring
  by hooks or loops

Stress in Helical Springs of Circular Wire:-

D - Mean dia of Spring coil
d - Dia. spring wire
n - no. of active coils
G - Modulus of rig.
W - Axial load
τ - Max. Shear stress

C - Spring index $= D/d$
p - pitch
δ - Deflection.



Twisting moment $T = W \times D/2$

$\& \; T = \pi/16 \, \tau_1 d^3.$

$$\tau_1 = \frac{8WD}{\pi d^3}.$$

τ_1 - Torsional Shear Stress.
τ_2 - direct Shear stress due to load W.

$$\tau_2 = \frac{W}{\text{X-Sect Area of wire}} \Rightarrow \frac{W}{\frac{\pi}{4}d^2} \Rightarrow \frac{4W}{\pi d^2}$$

Resultant shear stress:

$$\tau = \tau_1 \pm \tau_2 \Rightarrow \frac{8WD}{\pi d^3} + \frac{4W}{\pi d^2} \Rightarrow \frac{8WD}{\pi d^3}\left(1 + \frac{d}{2D}\right)$$

$$\tau = \frac{8WD}{\pi d^3}\left(1 + \frac{1}{2C}\right)$$

$$\boxed{\tau = K_S \frac{8WD}{\pi d^3}}$$

$K_S = 1 + \frac{1}{2C} \Rightarrow$ Shear Stress factor

we Neglected wire Curvature in above eq.

* It is noted that when the springs are subjected to static loads, the effect of wire curvature may be neglected.

S. Pipkyg

- If we consider both direct shear as well as curvature of wire, a Wahl's Stress factor (K) used.

$$\boxed{\tau = K \frac{8WD}{\pi d^3} \Rightarrow K \cdot \frac{8WC}{\pi d^2}}$$

$$\boxed{K = \frac{4C-1}{4C-4} + \frac{0.615}{C}}$$

also $\boxed{K = K_s \cdot K_c}$

K_c - Stress concentration factor due to curvature.

Deflection of Helical Springs of Circular Wire:-

Active Length $\Rightarrow l \Rightarrow$ length of one coil × No. of Active coil

$l \Rightarrow \pi D \times n$

Axial Deflection $\delta = \theta \times D/2$ — ①

θ Angular def. of wire when acted upon by torque "T"

$$\frac{T}{J} = \frac{\tau}{d/2} = \frac{G.\theta}{l}$$

$$\theta = T \cdot l / J \cdot G \qquad (J = \frac{\pi}{32} d^4)$$

$$\theta \Rightarrow \frac{(W \times \frac{D}{2}) \times \pi D . n}{\frac{\pi}{32} d^4 \times G} \Rightarrow \frac{16 W D^2 n}{G . d^4} \quad \text{— ②}$$

put in 1 we get

$$\boxed{\delta = \frac{8 W D^3 n}{G . d^4} \Rightarrow \frac{8 W C^3 n}{G . d}}$$

Stiffness of spring = $\boxed{k = \frac{W}{\delta} \Rightarrow \frac{G . d^4}{8 D^3 n} = \frac{G . d}{8 C^3 n}}$

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- Eccentric Loading

sof load on spring may be obtin by multiplying the Axial load (W) by factor $= \frac{D}{2e+D}$

e- eccentricity of load.

- Buckling of Compression Springs:

If L_f is more than tour times of mean or pitch dia-(D)

$$L_F \geq 4D$$

Wcr - Critical Axial load

$$W_{cr} = k \times K_B \times L_F$$

k = Stiffness

L_F = Free Length

K_B - Buckling factor depend upon L_F/D (Data Book)

To Avoid → used Central rod or Tube:

- Surge in Springs:-

When one end of a helical spring is resting on a rigid support & the other end is loaded suddenly, then all the coils of the spring will not suddenly deflect equally, because some time is required for the propagation of stress along the spring wire. A little consideration will show that in the begining, the end coils of spring in contact with applied load takes up whole of the deflection & then it transmits a large part of its deflection to the adjacent coils. In this way, a wave of compression propagates through the coils to the supported end from where it is reflected back to the detected end. This wave of compression travels along the spring indefinitely. If the applied load is fluctuating type as in the case of valve spring. So if the time interval b/w the load application is equal to the time required for the wave to travel from one end to other end, the resonance will occur. This results very large deflections of coils & very high stresses. & spring may fail. This phenomenon is called Surge.

⑥

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It has been found that the natural frequency of spring should be atleast twenty times of the frequency of application of a periodic load in order to avoid resonance with all harmonic frequencies up to twentieth order. the natural frequency for spring clamped b/w two plates is given by:

$$f_n = \frac{d}{2\pi D^2 n} \sqrt{\frac{6 \cdot G \cdot g}{\rho}} \quad \text{cycles/s}$$

d - wire dia G - Modulus of rig.
D - mean dia of coil g - Acc. due to gravity
n - number of active turns ρ - Density of wire material.

→ Surging in Spring may be eliminated by:
- Using friction dampers on the centre of coil so that the wave propagation dies out.
- By using spring of high natural frequency.
- By using spring having pitch of the coil near the ends different than at the centre to have diff. natural freq.

#
23.1 → A compression coil spring.

$D - 50\ mm$, $d = 5\ mm$ $n = 20$, $W = 500\ N$

$\tau = ?$

$C = D/d = 10$

Shear stress factor $K_s = 1 + \frac{1}{2C} = 1.05$

(Neg. curvat.) $\tau = K_s \frac{8 W \cdot D}{\pi \cdot d^3} \Rightarrow 534.7\ N/mm^2$ (MPa)

Q.23.2 - $d - 6\ mm$, $D_0 = 75\ mm$ $\tau = 350\ MPa$, $G = 84\ N/mm^2$

$W = ?$ & $\delta/n ?$

$D = D_0 - d \Rightarrow 69\ mm$, $C = D/d = 11.5$

Neg. curvature

$K_s = 1 + \frac{1}{2C} \Rightarrow 1.043$

$\tau = K_s \frac{8 W D}{\pi d^3}$

A $W = 412.7\ N.$

$\frac{\delta}{n} = \frac{8 W D^3}{G d^4}$

A $\Rightarrow 9.96\ mm$

Con. curvature

$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = 1.123$

$\tau = K\ 8WD/\pi D^3 \Rightarrow$ or $\boxed{\frac{8WC}{\pi d^2} \cdot K}$

$W = 383.4\ N.$

$\frac{\delta}{n} = \frac{8 W D^3}{G d^4}$

$\frac{\delta}{n} = 9.26\ mm$

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Energy stored in Helical Springs:

$$U = \frac{1}{2} W \cdot \delta$$

$$\tau = K \cdot \frac{8WD}{\pi d^3} \qquad W = \pi d^3 \tau / 8KD$$

$$\delta = \frac{8WD^3 n}{G \cdot d^4} \Rightarrow \frac{\pi \tau D^2 n}{K \cdot d \cdot G}$$

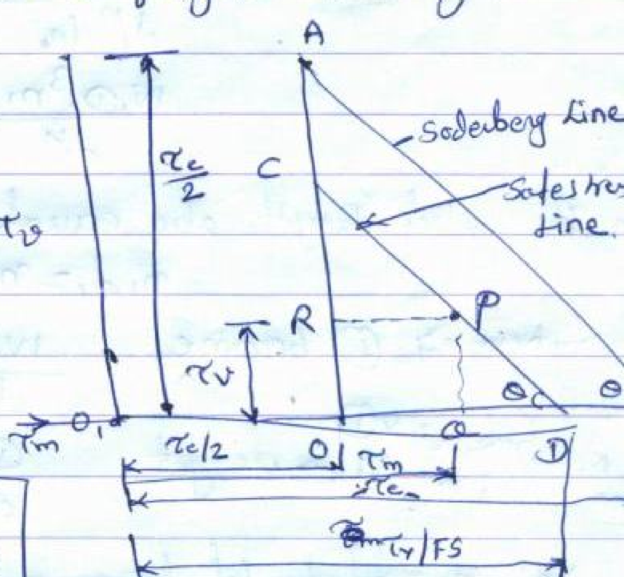
$$U = \frac{1}{2} \frac{\pi d^3 \tau}{8KD} \times \frac{\pi \tau \cdot D^2 \cdot n}{K \cdot d \cdot G}$$

$$\boxed{U \Rightarrow \frac{\tau^2}{4K^2 G} \underbrace{(\pi D \cdot n) \cdot \left(\frac{\pi}{4} d^2\right)}_{\text{volume}}}$$

$$\boxed{U = \frac{\tau^2}{4K^2 G} \cdot V}$$

Helical Springs Subjected to fatigue Loading:-

By modified Soderberg Line:-



Show:

$$\boxed{\frac{1}{F.S.} = \frac{\tau_m - \tau_v}{\tau_y} + \frac{2\tau_v}{\tau_e}}$$

$$\boxed{F.S. = \frac{\tau_y}{\tau_m - \tau_v + \frac{2\tau_v \tau_y}{\tau_e}}}$$

$$\tau_m = K_s \frac{8W_m \cdot D}{\pi d^3}$$

$$K_s = 1 + \frac{1}{2C} \quad \& \quad W_m = \frac{W_{max} + W_{min}}{2}$$

$$\tau_v = K \frac{8W_v D}{\pi d^3}$$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} \quad \& \quad W_v = \frac{W_{max} - W_{min}}{2}$$

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Concentric or Composite Spring:

→ To obtain greater spring force within space
→ To ensure the operation of a mechanism in the event of failure of one of the springs.

- Assume both spring have same material, then the maximum shear stress in both springs is same

i.e. $\tau_1 = \tau_2$

$$\frac{8 W_1 D_1 K_1}{\pi d_1^3} = \frac{8 W_2 D_2 K_2}{\pi d_2^3}$$

when $K_1 = K_2$

$$\boxed{\frac{W_1 D_1}{d_1^3} = \frac{W_2 D_2}{d_2^3}} \quad \text{--- (1)}$$

- If $\delta_1 = \delta_2$

$$\frac{8 W_1 D_1^3 n_1}{d_1^4 G} = \frac{8 W_2 D_2^3 n_2}{d_2^4 G}$$

$$\frac{W_1 D_1^3 n_1}{d_1^4} = \frac{W_2 D_2^3 n_2}{d_2^4} \quad \text{--- (2)}$$

- If solid length also equal

$$n_1 d_1 = n_2 d_2$$

then eq. (2) become $\quad \frac{W_1 D_1^3}{d_1^5} = \frac{W_2 D_2^3}{d_2^5}$ --- (3)

Now from (3) & (1)
i.e. 3/1 → we get $\quad \frac{D_1}{d_1} = \frac{D_2}{d_2} = C$ Spring Index --- (4)

i.e. springs should design such that spring Index will be same

from eq. (1) & (4) $\quad \left| \frac{W_1}{W_2} = \frac{d_1^2}{d_2^2} \right|$ --- (5)

Radial clearance b/w two springs $\quad c = \left(\frac{D_1}{2} - \frac{D_2}{2}\right) - \left(\frac{d_1}{2} + \frac{d_2}{2}\right)$ --- (6)

& usually it is taken as ⇒ $\frac{d_1 - d_2}{2}$ --- (7)

so from 6 & 7 $\quad \frac{D_1 - D_2}{2} = d_1$

from eq. (4) $\quad C d_1 - C d_2 = 2 d_1$

$$\boxed{\Rightarrow \frac{d_1}{d_2} = \frac{C}{C-2} \Leftarrow}$$

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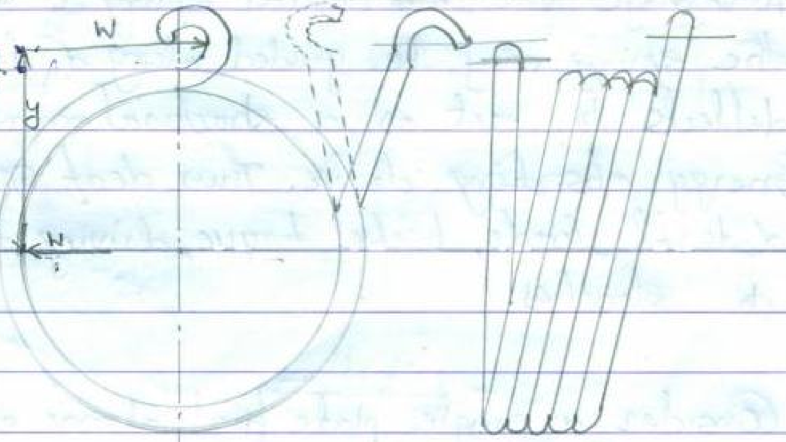
Unit Test No.	Date	Code No. / Subject	Roll No. & Name	Marks Obtained
		Machine Design	Spring Notes	Maximum Marks

Helical Torsion Spring.



made from round wire.
rectangular or square wire.
- The Ends are shaped to transmit torque.
- primary stress is Bending stress
- A little Consideration will show that the radius of curvature of coils changes when the twisting moment is applied.

show
$$\sigma_b = K\frac{32M}{\pi d^3} \Rightarrow K\times\frac{32\cdot W\cdot y}{\pi d^3}$$

$$K = \text{Wahl's factor} \Rightarrow \frac{4C^2-C-1}{4C^2-4C}$$

Now
$$\theta = \frac{M\cdot l}{E\cdot I} \Rightarrow \frac{M\times\pi D n}{E\times\pi d^4/64}$$

$$\theta \Rightarrow \frac{64 M D n}{E\cdot d^4}$$

C - Spring Index
M - Bending Moment $\Rightarrow W\cdot y$
W - Load
y - Distance W & Axis
d - Spring wire diameter
l = length of wire $\Rightarrow \pi D n$

Now
$$\delta \Rightarrow \theta\times y \Rightarrow \frac{64 M D n}{E d^4}\times y$$

In case of rectangular Spring: width - b & thickness t

$$\sigma_b = K\cdot\frac{6M}{t\cdot b^2} \Rightarrow K\frac{6W\cdot y}{t\cdot b^2}$$

$$K = \frac{3C^2-C-0.8}{3C^2-3C}$$

$$\theta = \frac{12\pi M D\cdot n}{E\cdot t\cdot b^3} \quad \&$$

$$\boxed{\delta = \theta y \Rightarrow \frac{12\pi M D\cdot n}{E\cdot t\cdot b^3}\cdot y}$$

For square Spring

$$\sigma_b = K\cdot\frac{6W\cdot y}{b^3}$$

$$\delta = \frac{12\pi M D\cdot n}{E b^4}\cdot y$$

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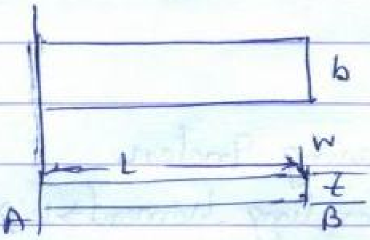
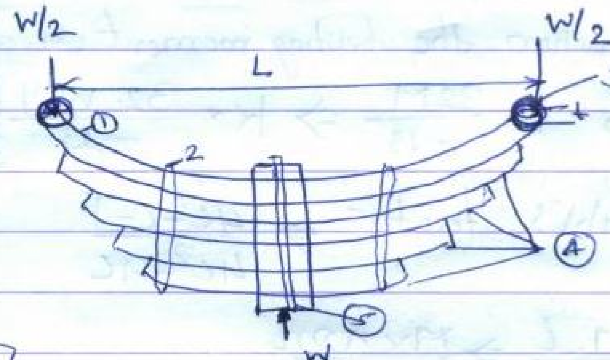
Laminated / Leaf Springs / Flat Springs:

- made out of flat plate
- Advantage over ~~leaf~~ Helical spring is that the ends of the spring may be guided along a definite path as it deflects to act as a structural member in addition to Energy absorbing device. Thus leaf spring may carry: lateral loads, brake torque, driving torque etc. in addition to shocks.

Consider a single plate fixed at one end & loaded at other end





t - thickness
b - width
L - Length
W - Load

$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$$

1. Master leaf
2. Metal Clips / Rebound
3. Eye to body & shackle
A - Additional Springs
5. U-Bolt

$M_{max} = W \cdot L$

$$Z = \frac{I}{y} \Rightarrow \frac{bt^3/12}{t/2} \Rightarrow \frac{bt^2}{6}$$

(Section modulus)

$$\sigma = \frac{M}{Z} \Rightarrow \frac{WL}{bt^2/6}$$

$$\boxed{\sigma = \frac{6 \cdot W \cdot L}{b \cdot t^2}}$$

Types → Semi Elliptical
Quarter —"—
Three Quarter —
Full Elliptical
transverse

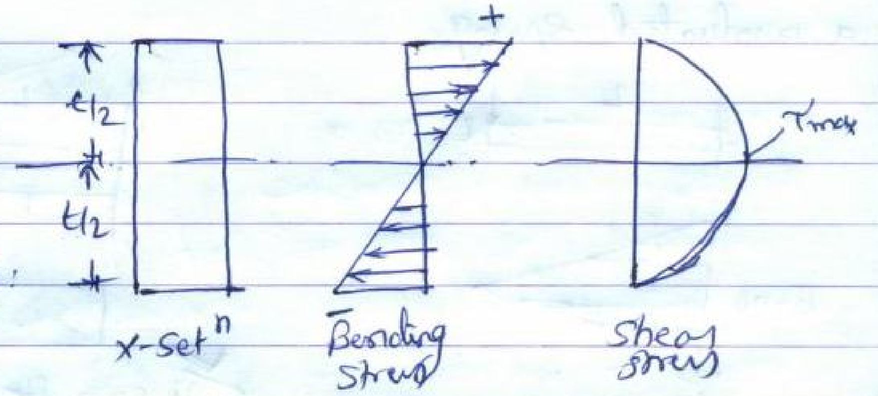
We know max deflection of cantilever with load W at end is

$$\delta = \frac{WL^3}{3 \cdot E \cdot I} \Rightarrow \frac{W \cdot L^3}{3E \times bt^3/12} \Rightarrow \frac{4WL^3}{E \cdot b \cdot t^3} \Rightarrow \frac{6WL}{bt^2} \times \frac{4}{6}\frac{L^2}{tE}$$

$$\boxed{\delta = \frac{2}{3}\frac{\sigma L^2}{E \cdot t}}$$

It is noted that due to bending moment top fibres in tension & bottom are in compression, but shear stress is zero at extreme fibres & max. at centre.

Hence for analysis, both stresses not to be taken into account simultaneously, we shall consider the bending stress only:



If the spring is not cantilever type but it is like a simply supported beam with length 2L & load 2W in the centre

Max. bending moment in the centre

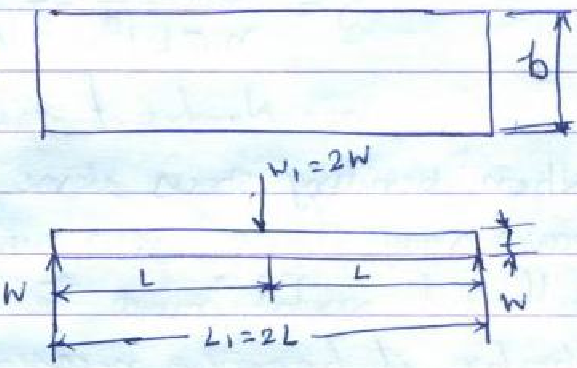
$$M = W.L \;; \quad M = \frac{W_1}{2} \times \frac{L_1}{2} \Rightarrow \frac{2W}{2} \times \frac{2L}{2}$$

Section Modulus

$$Z = bt^2/6$$

Bending Stress $\sigma = \frac{M}{Z} \Rightarrow \frac{W.L}{bt^2/6}$

$$\boxed{\sigma = 6WL/b.t^2}$$



- We know max. deflection of simply supported beam load at centre:

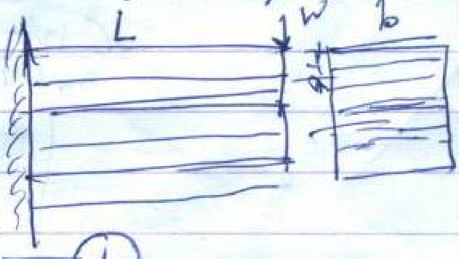
$$\delta = \frac{W_1 L_1^3}{48EI} \Rightarrow \frac{2W.(2L)^3}{48EI} \Rightarrow \frac{W.L^3}{3EI} \Rightarrow \frac{WL^3}{3E.bt^3/12} \Rightarrow \frac{4WL^3}{E.bt^3}$$

- From above eq. we see that a spring such with length 2L & load in centre 2W, may be treated as double cantilever

- If the plate of cantilever is cut into a series of n strips of width b & placed below each other then

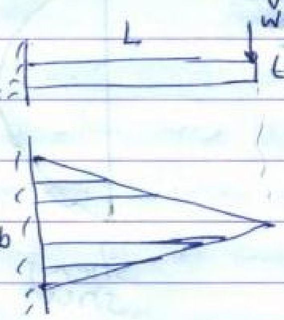
$$\sigma = \frac{6.W.L}{n.b.t^2}$$

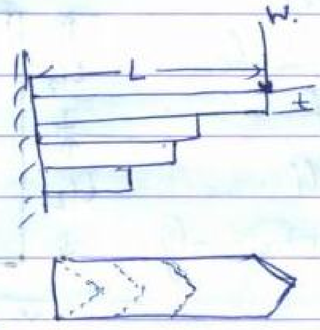
$$\delta = \frac{4WL^3}{Eb.t^3 n} \Rightarrow \frac{2\sigma L^2}{3E.t} \quad \text{--- (1)}$$



The above relations give the stress & deflection of leaf spring of uniform x-sect^n. Thus stress at such that a spring is max at support.

If a Δ plate is used, as the stress will be uniform throughout. If this Δ plate is cut into strips of uniform width and placed on below the other to form a graduated spring.





~~It can be proved that the deflection at the load point of Δ plate given by:~~ $\cancel{\delta = WL^3/2EI \Rightarrow WL^3/2E(\tfrac{1}{12}nb\,t^3)}$

$$\sigma = \frac{6 \cdot W \cdot L}{n \cdot b \cdot t^2}$$

$$\delta = \frac{6 \cdot W \cdot L^3}{n \cdot E \cdot b \cdot t^3} \Rightarrow \frac{\sigma \cdot L^2}{E \cdot t} \quad \text{--- (2)} \Rightarrow \frac{6WL^3}{nE \cdot b \cdot t^3}$$

n = Number of graduated leaves

When bending stress alone is considered, the graduated leaves may have zero width at the loaded end. But sufficient metal must be provided to support the shear therefore it becomes necessary to have one or more leaves to uniform X-sec^n extending clear to the end. We see from equations ① & ② that for the same deflection, the stress in the uniform X-sec^n leaves (i.e. full length leaves) is 50% greater than in the graduated leaves, assuming that each spring elements deflects according to its own elastic curve. If the suffix F & G are used to indicate the full length & graduated leaves, then,

$$\delta_1 = \delta_2 : \frac{2}{3}\frac{\sigma_F L^2}{E \cdot t} = \frac{\sigma_G L^2}{Et}$$

$$\sigma_F = \frac{3}{2}\sigma_G \qquad \boxed{\sigma_G = \frac{2}{3}\sigma_F}$$

$$\frac{6W_F L}{n_F b \cdot t^2} = \frac{3}{2}\left[\frac{6 \cdot W_G L}{n_G b t^2}\right]$$

or

$$\frac{W_F}{W_G} = \frac{3}{2}\frac{n_F}{n_G} \quad \text{--- (3)}$$

Add 1 Both side $\frac{W_F}{W_G} + 1 = \frac{3n_F}{2n_G} + 1 \Rightarrow \frac{W_F + W_G}{W_G} = \frac{3n_F + 2n_G}{2n_G}$

$$W_G = \left(\frac{2n_G}{3n_F + 2n_G}\right)W. \quad \text{--- (4)}$$

W = Total load on spring = $W_G + W_F$

W_G = Load taken up by graduated leaves

W_F - Load taken up by full length leaves.

Again from eq. (3) $\frac{W_G}{W_F} = \frac{2n_G}{3n_F}$ add 1 both side.

$$W_F = \left(\frac{3n_F}{2n_G + 3n_F}\right) W. \quad \text{— (5)}$$

Bending stress for full length leaves

$$\sigma_F = \frac{6 W_F L}{n_F b \cdot t^2} \Rightarrow \frac{6 \cdot L}{n_F b \cdot t^2}\left[\frac{3n_F}{2n_G + 3n_F}\right] W.$$

$$\boxed{\sigma_F = \frac{18 WL}{b \cdot t^2 (2n_G + 3n_F)}} \quad \text{— (6)}$$

& $\sigma_F = \frac{3}{2}\sigma_G; \quad \sigma_G = \frac{2}{3}\sigma_F.$

$$\sigma_F = \frac{12 W \cdot L}{bt^2 (2n_G + 3n_F)} \quad \text{— (7)}$$

The deflection of full length & graduated leaves are given by eq. (1)

$$\delta = \frac{2 \sigma_F L^2}{3 \cdot E \cdot t} \Rightarrow \frac{2L^2}{3E t}\left[\frac{18 W \cdot L}{b \cdot t^2 (2n_G + 3n_F)}\right]$$

$$\boxed{\delta = \frac{12 W \cdot L^3}{E \cdot b \cdot t^3 (2n_G + 3n_F)}} \quad \text{— (8)}$$

Nipping of Leaf Springs:- / Stresses due to Support Hinges.

- ~~As discussed in the previous section, the stresses in extra full~~

The master leaf of laminated spring is hinged to the supports. The supports forces includes, stress due to longitudinal forces & stresses arising due to possible twist. Hence the master leaf is more stressed compared to other graduated leaves.

So Method of reducing additional stresses could be:

A. Master leaf is made by stronger material than others.

B. Master leaf is made thinner than the other leaves as $\sigma_b \propto 1/t^2$.

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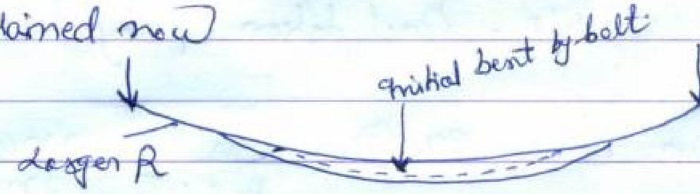
6. Another common practice is to increase the radius of curvature of the master leaf than next leaf.

The last one is explained now



The master leaf has a larger radius of curvature compared to the additional leaf that is placed below so a gap will be created b/w the two leaves as indicated in figure.

← Nipping →

Now a initial bent is created during assembly by bolt therefore some amount of compressive stress will be produced at the inside of curvature at master leaf. Similarly at the outside curvature at master leaf tensile stress will be produced. Both these are initial in master leaf. However by such bolt tightening the additional leaf that is placed beneath the master leaf has a tendency to flatten out & as a result the stress pattern of additional leaf will be reverse of that of master leaf. i.e. tensile stress is at inner curvature & compressive at outer curvature. Hence when spring is loaded, for both the master leaf & additional leaf tensile stress will be produced at the inner curvature & compressive at outer curvature. Therefore due to opposite nature of initial stress & loading stress, the master leaf will be experience lesser stress on both the surfaces. However due to same nature of initial stress & loading the additional leaf is stressed more compared to master leaf. But it is to be noted that the higher stress on additional leaf is actually shared b/w all other leaves than master leaf. This practice of stress relief in the master leaf is known as Nipping of Leaf Spring. As a matter of fact, all the leaves do have certain amount of nipping, so that there will be gaps between the leaves as a result the stresses will be uniformly distributed & accumulated.